Asymmetric Fission from Egg Shape Deformation and Comparison of Coulomb and Surface Energies for Fission Fragment Mass Combinations

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Abstract - Asymmetric fission from an egg shape deformation is studied for ^{235}U nucleus. For neck formation in the fissioning process, the place of least curvature of deformed nucleus is taken into account. Geometry of an egg shape is expressed in terms of its major and minor axes, and variation of Coulomb and surface energies studied. In the symmetric and asymmetric fission fragment mass combinations, significant variation of about 4 to 10 MeV in Coulomb energy is observed which can affect the fission barrier height. For the asymmetrically deformed nucleus like an egg, the shell effects are discussed. It is found that parity is not a good quantum number for asymmetric shape hence the asymmetric shape like an egg is supposed to evolve at the crossing of opposite parity levels.

Keywords — Fission, Asymmetric, Egg shape, geometry, Barrier, Coulomb, Surface, Shell effects

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1 INTRODUCTION

NUCLEAR fission was discovered around three quarters of a century ago in which the splitting of a heavy nucleus into various fragments was observed [1-3]. The general features of fission reactions were well understood both experimentally and theoretically but still it continued to throw up new challenges. One of the most persistent puzzles in the fission process is the asymmetric mass distributions of fission fragments. Involvement of very large number of nucleons and both macroscopic and microscopic features are expected to influence the fission phenomena [3]. Macroscopic features are basically of classical nature which largely determines the energetics of the fission phenomena. The liquid drop model [3-6] is a very well established classical model which describes the deformation process in terms of nuclear surface and Coulomb energy and corresponding fission barrier due to competition between surface and Coulomb energies [5-6]. On the other hand, asymmetric mass distribution of fission fragments pointed towards the more microscopic quantum effects [7-10]. These quantum effects describe the distribution of nucleons in the nuclear shells for a deformed potential.

2 THEORETICAL DESCRIPTION

Shape changes involved in the transition from a single spherical nuclear system to two separated daughter fragments is considered to be as a slow process. In the fissioning process however starting from a spherical shape, at some point of deformation the nucleus has to assume an asymmetrical shape in the fissioning direction. Some common deformation shapes [3, 11-13] are like ellipsoid, dumbbell, pear shape etc. One

of the simplest asymmetrical shapes is like an egg which is normally not studied. This egg shape is obtained basically from two hemi - ellipsoids of revolution about the major axes – a_1 and a_2 with equal minor axis - b, by joining the equal circular faces as seen in fig.1. With increasing elongation in the fissioning direction the major axes - a_1 and a_2 increases and corresponding minor axis - b decrease so as to conserve the total volume.
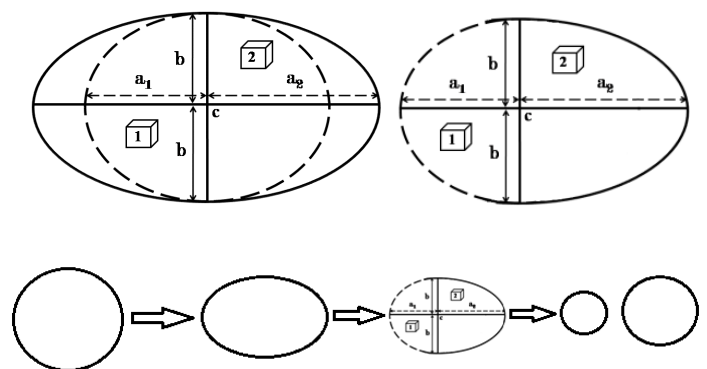
Fig.1: Schematic of an asymmetric egg shape (with minor axis b, and major axes a_1 and a_2) and deformation process via an egg shape. 1 and 2 are the two point charges within egg volume.

Fission occurs when the joint is snapped as a result of the total surface energy increasing beyond the total surface energy of the spherical shape. In the fissioning process it is important that the cleavage starts in this proceedings of elongation at a point where the surface curvature is least and

it is at the joint of the two hemi-ellipsoids.

3 NUMERICAL CALCULATIONS AND RESULTS

In the present paper, the numerical calculations are performed for evaluation of the Coulomb and surface energies for the nearly symmetric and highly asymmetric mass combinations of fission fragments of ^{235}U nucleus. The Coulomb energy is evaluated by performing six-dimensional integration as:

$$E_C = -a_c \int dx_1 dy_1 dz_1 \int dx_2 dy_2 dz_2 \frac{\rho}{r_{12}}$$

Where

$$a_c = \frac{3e^2}{20\pi\varepsilon_0 r_0} = 0.708 MeV \quad \text{and} \quad r_0 = 1.22 \text{ fm}$$

The surface energy however has an analytic form as:

$$E_s = a_s \left[2\pi b^2 + \pi \sum_{i=1}^{2} b \left[\frac{a_i^2}{\sqrt{a_i^2 - b^2}} cos^{-1}\left(\frac{b}{a_i}\right) \right] \right]$$

Where

$$a_s = 16.80 \text{ MeV}$$

And surface area is defined as

$$Surface area = \left[2\pi b^2 + \pi \sum_{i=1}^{2} b \left[\frac{a_i^2}{\sqrt{a_i^2 - b^2}} cos^{-1}\left(\frac{b}{a_i}\right) \right] \right]$$
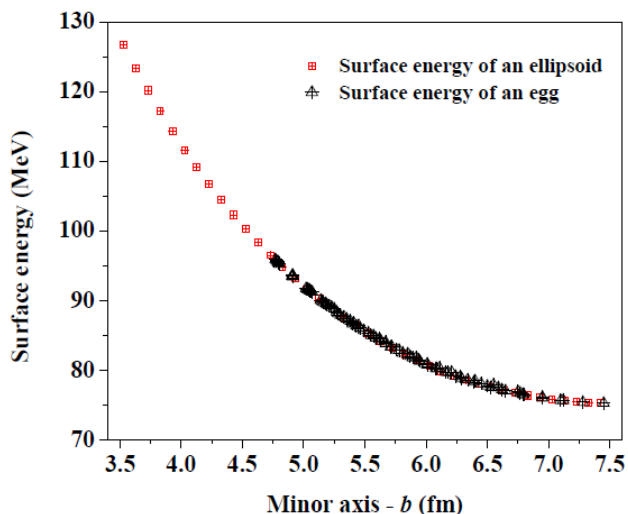
beginning of the deformation process rather it starts when the volume conservation, cleavage point ($b < a_1 < a_2$) and a_1 / a_2 conditions are satisfied simultaneously. However this occurs only after slight deformation has taken place. Variation of surface and Coulomb energy as a function of minor axis - b, for symmetric to asymmetric fission into various fragments mass combinations (as 80 + 155, 85 + 150 to 115 + 120) is shown in fig. 2 and fig. 3 respectively. The value of parameter - b decreasing from the sphere on these plots corresponds to the point where asymmetry starts developing and the last smallest value is the value where the surface energy equals that of the fragments (the saddle point). Comparison of surface energy between symmetric and asymmetric fission fragment mass combinations is shown in fig. 2. As a function of minor axes - b, the surface energy of symmetrical and asymmetrical mass combinations is seen to overlap and behaves similarly. Observed linear relation between the ratio of major axes a_1 / a_2 and minor axis - b is represented in fig. 4.
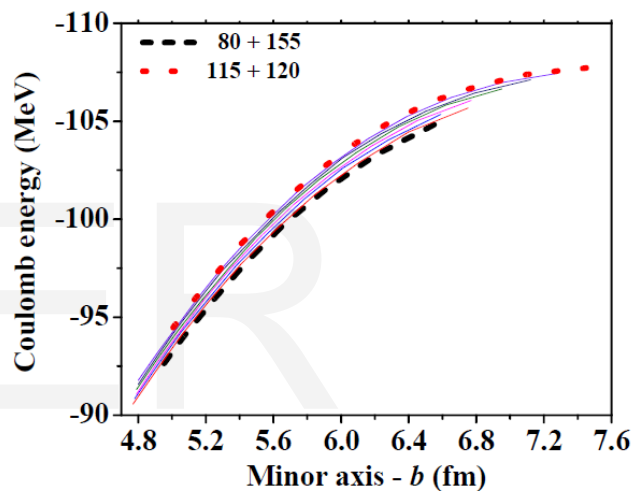

Fig. 3: Coulomb energy of fission fragments mass combinations as a function of minor axis - b.

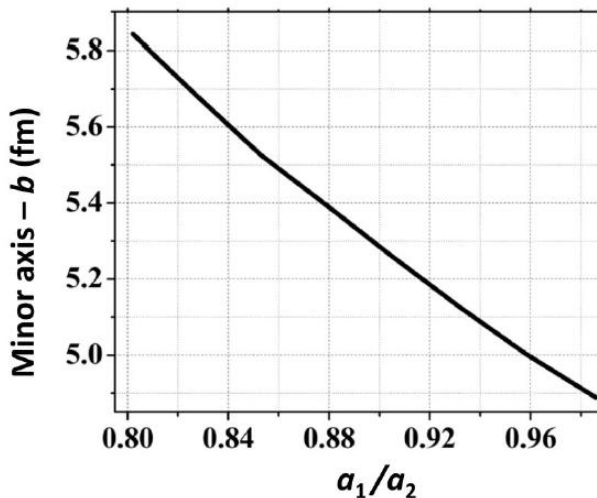**Fig. 2:** Surface energy of ellipsoid and an egg shaped nucleus.

Fig. 4: Variation of ratio of major axes a_1/a_2 with minor axis – b.

It is found that, final asymmetrical shape of an egg does not start at the

Most importantly from figure - 3, it is found that, the Coulomb energy for

asymmetric fragment mass combinations is seen to decreases from that of the symmetric fission fragments. Around 4 to 10 MeV energy differences are observed in between asymmetric and symmetric fission fragment mass combinations and this behavior can affect the fission barrier height. This also indicates that large amount of energy shell corrections are required for asymmetric deformations.

On the other hand, geometry of the egg shape in terms of combination of two hemi- ellipsoid is expressed in terms of major and minor axes and deformation parameter - ε [14] as;

$$a_1 = R(1 + \varepsilon_{11})$$
(Along major axis - a_1)

$$b = R/\sqrt{1 + \varepsilon_{12}}$$
and

$$a_2 = R(1 + \varepsilon_{21})$$
(Along major axis - a_2)

$$b = R/\sqrt{1 + \varepsilon_{22}}$$

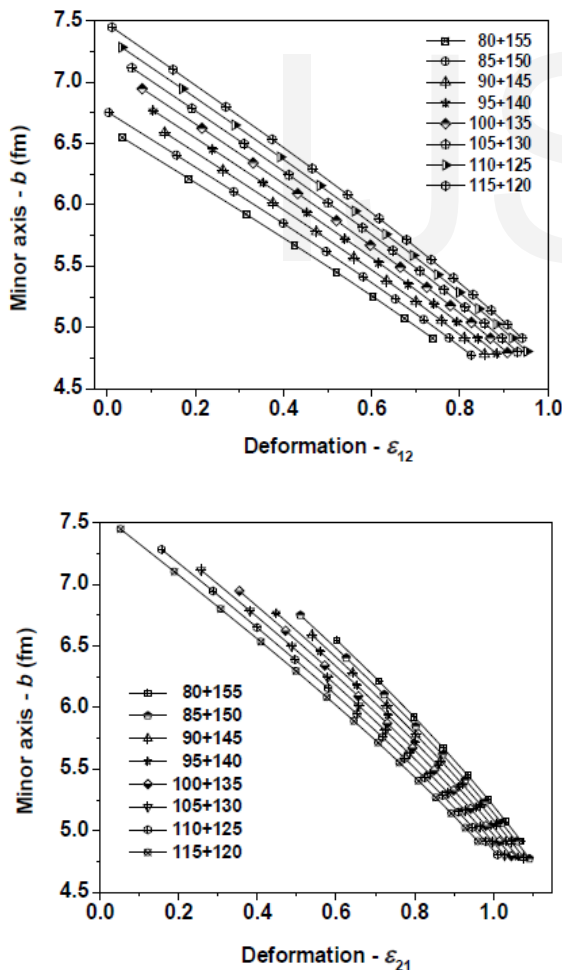Where $R = 7.53$ fm (Radius of spherical nucleus ^{235}U)

Fig. 5 (A & B): Variation of minor axis – b with deformation parameter ε_{12} and ε_{21}.

Here ε_{12} and ε_{21} have the same values. The relation of these deformation parameters with the minor axis - b is shown in fig. 5A and fig. 5B respectively.

The mass asymmetry in fission is greatly influenced by the distribution of nucleons in the nuclear shells. It is reported that the liquid drop energy surface is stable with respect to the asymmetric distortions [3, 15-16]. For large P_2 distortions, the liquid drop energy surface becomes quite soft with respect to distortions that involve a certain combinations of P_3 and P_5 deformations [15-16]. It is thus possible that at certain deformations, shell effects might overcome the liquid drop stability with respect to asymmetric distortions [16].

To study the cause of asymmetric distortions, the Nilsson diagram [17] of deformed nucleus is employ. In the Nilsson diagram it is found that the asymmetry can be start within 0.0 to 0.3 of the range of deformation - ε and observed that there are some points where the energy shells those having same spin (like 1/2, 3/2, 5/2, 7/2, 9/2 etc.) but opposite parity (solid and dotted lines) cross each other. These are the points (the point of crossing of solid and dotted lines which have same spin but opposite parity) where mixed parity exists and asymmetries being developed in the form of asymmetric shape like an egg. These points are studied in the Nilsson diagram of deformed nucleus for Z > 82 and also for N ≥ 126 respectively.

4 SUMMARY AND CONCLUSION

The deformations and fission fragment mass asymmetry is discussed by combining both macroscopic and microscopic methods. Asymmetric fission via an egg shape deformation is studied. One important result here is that the cleavage starts at that point where the surface curvature is least and this is the place of joint of two hemi-ellipsoids. The present perception says that the lesser eccentricity ellipsoid, a_1/b part of the egg become the smaller fragment while the bigger eccentricity part; a_2/b become the larger fragment. The prevailing perception in the available literature does not describe the phenomena of asymmetric fission via the formation of an egg shape and they tend to describes the asymmetric fission through some neck formation with one big blob on one side while a smaller one on the other side. So far this aspect of P_3 distortions for asymmetric fission has not been highlighted for this egg shaped deformation and present work indicates that higher amount of shell corrections to be applicable in the asymmetric fission. In the Nilsson diagram it is found that the asymmetry starts at the points where the energy shells of same spin but opposite parity (solid and dotted lines) cross each other. These are the points where mixed parity exists and asymmetry being developed in the form of asymmetric shape like an egg.

ACKNOWLEDGEMENT

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